MODELING A STRIPLINE FERRITE PHASE SHIFTER

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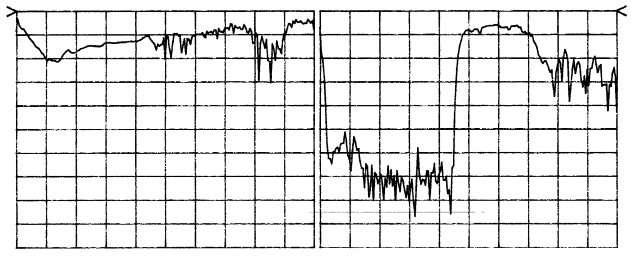
Abstract

A model is proposed for the behavior of transmission lines on ferrite substrates. I believe this is the first time a model has correctly described the absorption below the gyromagnetic resonant frequency corresponding to the magnetization at saturation, or $\gamma 4\pi M_s$. This new model should allow accurate prediction of impedances and propagation constants for frequencies operating close to this cutoff frequency, permitting more accurate transmission lines and transformers to be designed. Comparison with measured data from a low-temperature co-fired (LTCC) ferrite stripline phase shifter is given.

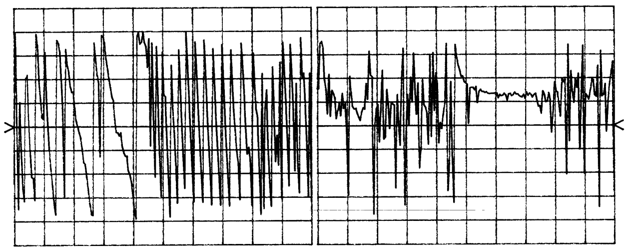
Introduction

Microwave ferrite devices often operate with an applied magnetic field, so that the ferrite is at or above magnetic saturation. That is, the domains are fully aligned in a particular direction. In this case, the Polder tensor model of the ferrite gives reasonably accurate predictions of the behavior of the device. In the standard approach[1], the permeability of a ferrite magnetized in the z direction is taken to be a tensor $\underline{\mu}$ with $\mu_{xx}=\mu_{yy}=\mu$, $\mu_{zz}=1$ and $\mu_{xy}=-\mu_{yx}=\kappa$, where $\mu=1+\omega_o\omega_m/(\omega_o^2-\omega^2)$, $\kappa=\omega\omega_m/(\omega_o^2-\omega^2)$, and $\omega_o=\gamma H_{int}$, $\omega_m=\gamma 4\pi M_s$. If the ferrite is biased just at saturation, so that the internal field $H_{int}=0$, these reduce to $\mu=1$ and $\kappa=-\omega_m/\omega$. Solution of Maxwell's equations for TEM propagation produces an effective permeability μ_{eff}. If the DC magnetization is perpendicular to the RF magnetic field, then $\mu_{eff}=(\mu^2-\kappa^2)/\mu=1-(\omega_m/\omega)^2$. If the DC magnetization is aligned parallel to the RF magnetic field, then $\mu_{eff}=1$. These formulas were used to predict the performance of an earlier stripline phse shifter based on the idea of re-orienting the remanent magnetization[2].

If we measure the transmission and reflection coefficients of a microstrip or stripline on an unmagnetized ferrite substrate, a characteristic absorption band is found, between nearly zero frequency and $\gamma 4\pi M_s$. This is commonly referred to as a "low-field" loss. Figure 1 shows an example of this behavior in a stripline with $4\pi M_s$=4700 gauss, and a nominal impedance of 25 Ω (chosen to reduce conductor loss), connected between 50 Ω source and load impedances. Note that this data includes the effects of impedance matching sections and bias tees attached to each end of the transmission line. This accounts for some of the suckouts and high loss at the highest frequencies.



Return Loss and Insertion Loss (10 dB/div)



Insertion Phase (45 /div), Time Delay (1 nsec/div)

Figure 1. Measured response of 2.3" long LTCC ferrite stripline, 0.045-26.5 GHz.

Circuit Model

The insertion phase and time delay in figure 1 have a behavior similar to that of standard waveguide: the phase velocity goes to infinity and the group velocity goes to zero at cutoff. Figure 2 shows a lumped element equivalent circuit for a lossless TEM transmission line, L and C being the inductance and capacitance per unit length. The characteristic impedance Z_c is equal to $\sqrt{(j\omega L/j\omega C)}=\sqrt{(L/C)}$ or $\sqrt{(\mu_{eff}/\varepsilon)}$. The propagation constant is $\sqrt{(j\omega L j\omega C)}=j\omega\sqrt{(LC)}=j\omega\sqrt{(\mu_{eff}\,\varepsilon)}$, giving a phase velocity $v_p=1/\sqrt{(LC)}=1/\sqrt{(\mu_{eff}\,\varepsilon)}$, which is just $c/\sqrt{\varepsilon_r}$ if $\mu_{eff}=\mu_o$. We also have $L=Z/v_p$ and $C=1/(Z\ v_p)$.

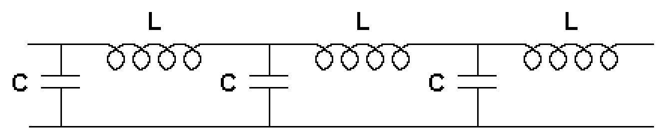


Figure 2. Lumped element circuit model of transmission line with $Z=\sqrt{(L/C)}$.

This model can be extended to simulate a waveguide having a cutoff frequency ω_c by adding a series capacitance (1/capacitance per unit length) $C_c=1/(\omega_c^2\,L)$. Now the phase velocity becomes $v_{pc}=1/\sqrt{(L(1-(\omega_c/\omega)^2)C)}$, which goes to infinity at cutoff, and the group velocity becomes $v_{gc}=(1-(\omega_c/\omega)^2)/\sqrt{(LC)}$, which goes to zero at cutoff.

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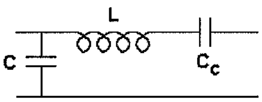


Figure 3. Lumped element model with cutoff at $1/2\pi\sqrt{(LC_c)}$.

When we say the ferrite is not magnetized, we really mean that there is no *net* magnetization in the overall sample. In reality, the ferrite is made up of a large number of small domains, each of which is fully magnetized at $4\pi M_s$. Each domain is randomly oriented, so the net effect is $\underline{\mathbf{M}}=0$. An individual domain is subjected to the field produced by all the domains around it, which can range from 0 to $4\pi M_s$. A magnetized domain exhibits a ferromagnetic resonance frequency $\omega_o=\gamma H_{int}$. Each domain is therefore an individual resonator at some frequency between 0 and $\gamma 4\pi M_s$. Now imagine a stripline in which the substrate comprises a large number of resonant absorbers. Since all the RF magnetic field is contained within this volume of absorbers, the equivalent circuit in figure 4 emerges. The series inductance L is divided into M inductances in parallel, each having inductance M·L. Each inductance has a series resonating capacitance $C_k=1/(\omega_k^2\, M{\cdot}L)$, where $0<\omega_k<\gamma 4\pi M_s$. For simplicity, assume that the coupling to each resonator is equal. A series resistance R_k represents the absorption of energy in each resonator. This is a critical feature of the analysis, and differs from the cutoff waveguide model, which assumes a purely reactive effect at cutoff. R_k can be estimated from the Q of the resonance: $R_k=\omega_k ML/Q_k$. Note that this is not the resistance of the strip, which could be modeled by adding a separate series resistance to the model.

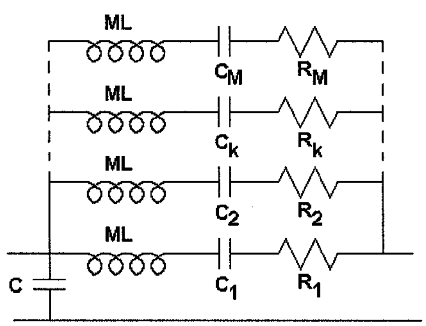


Figure 4. Model of transmission line with M resonant absorbers.

The characteristic impedance of this transmission line is $Z_c=\sqrt{(Z_{series}Z_{shunt})}=\sqrt{(Z_{shunt}/Y_{series})}$, where $Y_{series}=\Sigma Y_k$, and $Y_k=1/Z_k$.

$$Y_{series}=\sum_{k=1}^{M}\frac{1}{j\omega ML\left[1-\left(\frac{\omega_k}{\omega}\right)^2-j\frac{\omega_k}{\omega}\frac{1}{Q_k}\right]}$$

The domains are quite small, say 100 µm = 10^{-2} cm across, so any reasonably sized ferrite sample will contain a large number of individual resonators, e.g. M=(1 cm x 1 cm x 0.1 cm)/$(10^{-2}\ \text{cm})^3$ = 10^5. The sum can therefore be approximated as an integral,

$$Y_{series}:=\frac{1}{j\omega L}\cdot\frac{1}{M}\cdot\int_0^{M+1}\frac{1}{1-j\cdot\frac{\omega_k}{\omega_m}\cdot\frac{\omega_m}{\omega}\cdot\frac{1}{Q_k}-\left(\frac{\omega_k}{\omega_m}\cdot\frac{\omega_m}{\omega}\right)^2}\,dk$$

For simplicity, assume that Q_k is independent of k. Evaluating the integral, and inverting the admittance gives the series impedance term:

$$Z_{series}:=\frac{j\omega_m\cdot L\cdot\sqrt{4-\frac{1}{Q^2}}}{\left[\ln\left[\frac{\sqrt{4-\frac{1}{Q^2}}+2\cdot\frac{\omega_m}{\omega}+\frac{j}{Q}}{\sqrt{4-\frac{1}{Q^2}}-2\cdot\frac{\omega_m}{\omega}-\frac{j}{Q}}\right]-\ln\left[\frac{\sqrt{4-\frac{1}{Q^2}}+\frac{j}{Q}}{\sqrt{4-\frac{1}{Q^2}}-\frac{j}{Q}}\right]\right]}$$

For $Q\to\infty$ and $\omega>>\omega_m$, this expression reduces to the usual value $j\omega L$. For $\omega<\omega_m$, the real part of the argument of the log term is negative, and Z_{series} becomes complex instead of pure imaginary, even if Q is very large. This produces a high attenuation per unit length and a complex characteristic impedance below ω_m.

Figure 5 shows the variation in the characteristic impedance and propagation constant resulting from this model. The effective series inductance drops as the frequency approaches ω_m from above. Another way of looking at this is that the effective permeability drops to zero at ω_m, and becomes complex below ω_m. The long-dash curve is the propagation constant for a nonmagnetic substrate with the same dielectric constant.

If this type of transmission line is connected between source and load impedances $Z_s=Z_l=50\ \Omega$, with $Z_{nom}=25\ \Omega$, we predict the response shown in figure 6, assuming Q=157. (We will see later that the results are not particularly sensitive to Q, as long as a reasonable value is assumed.)

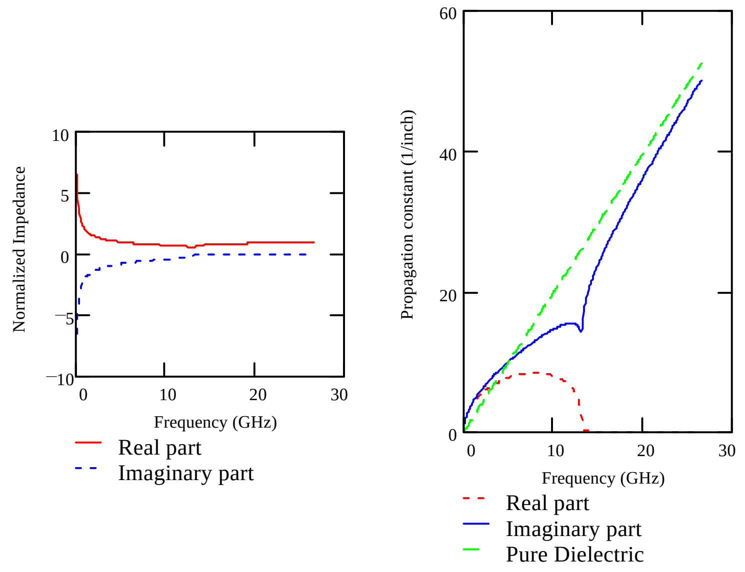


Figure 5. Normalized transmission line characteristic impedance and propagation constant in ferrite-filled stripline.

The calculated results agree with experiment in these key respects:

1. high attenuation at $\omega<\omega_m$
2. peaking of time delay just above ω_m
3. strong ripple in the passband above ω_m
4. smooth return loss in absorption band
5. gradual increase in return loss as frequency goes below ω_m
6. abrupt decrease in return loss as frequency approaches zero

High attenuation is seen below ω_m because of the band of resonant absorbers. Peaking of the time delay is due to the decrease in group velocity, or equivalently, the increase in phase velocity, as the frequency approaches ω_m from above. The ripple in the passband is due to impedance mismatch between the source and load impedances and the lower-than-normal transmission line impedance. The smooth return loss in the absorption band is due to high absorption in the ferrite which attenuates any load mismatch. The return loss improves as we move lower in frequency because the real part of the impedance rises toward the nominal value. The return loss drops abruptly at the lowest frequency because the impedance becomes quite large there.

Figure 7 shows the effect of varying the assumed value of Q. The insertion loss below ω_m is so high that the input impedance is essentially equal to Z_c, regardless of the termination impedance. Z_c appears to be relatively independent of Q. An estimate of the Q is $\omega/K\gamma\Delta H$, where K is a coupling coefficient. The effect of changing the value of Q may be thought of in terms of varying either the ferrite Q (~1/linewidth), or the coupling between the strip and the resonators. The absorption band persists in the measured response in both RESET and SET states. Above ω_m, the insertion loss decreases with higher Q because the resistances R_k are lower.

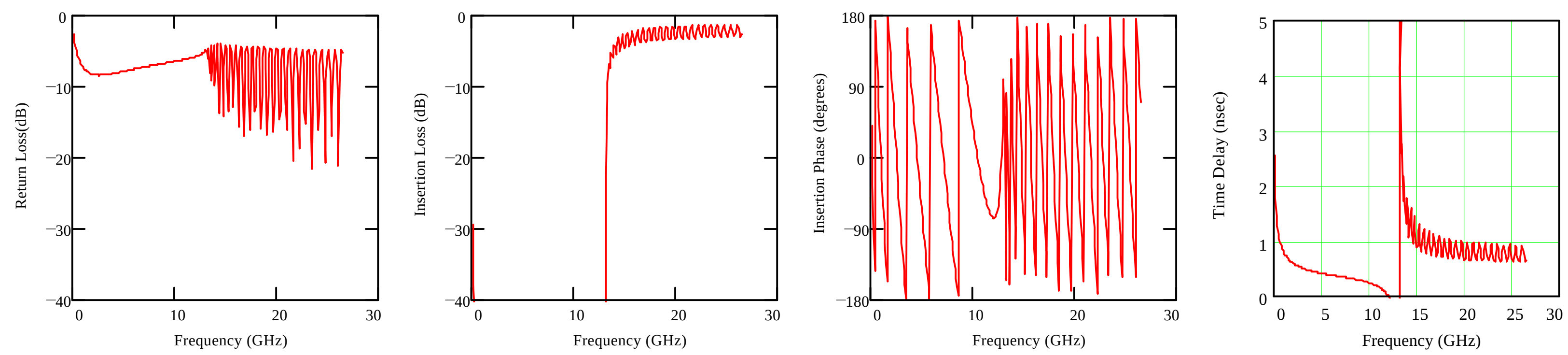
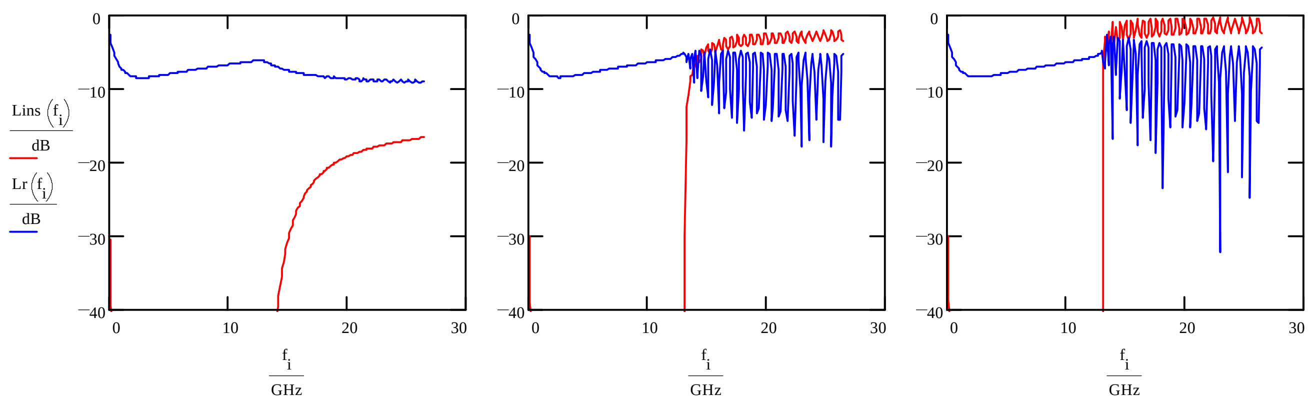


Figure 6. Calculated return loss L_r, insertion loss L_{ins}, insertion phase Φ, and group delay $\tau = -d\Phi/d\omega$ in 2.3" long ferrite-filled stripline.



Q=10 Q=100 Q=1000

Figure 7. Calculated response for different values of Q.

Phase Shift

For transmission phase calculations above ω_m, we can use a simplified version of the integral in Z_{ser}. In the limit of $Q \to \infty$, and for $\omega > \omega_m$, the expression for the effective permeability simplifies to

$$\mu_{simple} = 2\,(\omega_m/\omega)/\ln((\omega + \omega_m)/(\omega - \omega_m))$$

This expression is in close numerical agreement with Schlömann's expression[3] for $\omega > \omega_m$:

$$\mu_{Schl}(f) := \frac{1}{3}\cdot\left[1 + 2\cdot\sqrt{\left|1 - \left(\frac{f_s}{f}\right)^2\right|}\right]$$

The experimental data show that in the remanent state, the absorption band is always present, and the cutoff frequency is nearly constant, regardless of the orientation of the magnetization. In the LTCC stripline phase shifter, the RESET condition is obtained by pulsing a current through the center conductor, latching the DC magnetization essentially parallel to the RF magnetic field. This gives very little coupling between the RF and the ferrite, so the effective permeability is just μ_{simple}. The SET condition is obtained by pulsing a current through a coil around the ferrite. This orients all the domains more or less parallel to the center conductor, or perpendicular to the RF magnetic field. The degree of alignment corresponds to the remanent value of B_r. This results in strong coupling not just to individual domains, but to the entire sample of ferrite, which now has a more or less uniform precession resonance at $\omega = \gamma H = \gamma B_r$. Since this is a single resonance, it is plausible that the permeability is decreased by $(\gamma B_r/\omega)^2$, just as the cutoff frequency in waveguide reduces the propagation constant by $(\omega_c/\omega)^2$. The following function is plotted in figure 8:
$\mu_{eff(set)} = \mu_{simple} - (\gamma B_r/\omega)^2$. Notice that the cutoff frequency does not move significantly until B_r exceeds about $0.7\ 4\pi M_s$. Figure 9 shows good agreement between the calculated and measured phase shift versus B_r for experimental devices having B_r values between 1500 and 2650 gauss using the formula

$$\Delta\Phi(f) = 360^\circ\, f\, \sqrt{\varepsilon_r}\, \{\sqrt{\mu_{simple}} - \sqrt{[\mu_{simple} - (\gamma B_r/\omega)^2]}\ \} / c.$$

Although this is formally somewhat different from an expression given by Green and Sandy[4], the numerical results are quite similar.

CONCLUSIONS

An expression for an effective permeability has been derived which predicts the impedance and propagation constant for striplines on ferrite substrates having a remanent magnetization. Phase shift can be accurately predicted as a function of frequency and magnetic properties of the substrate. Substantially more phase shift was found in an LTCC stripline phase shifter than was expected from earlier published work.

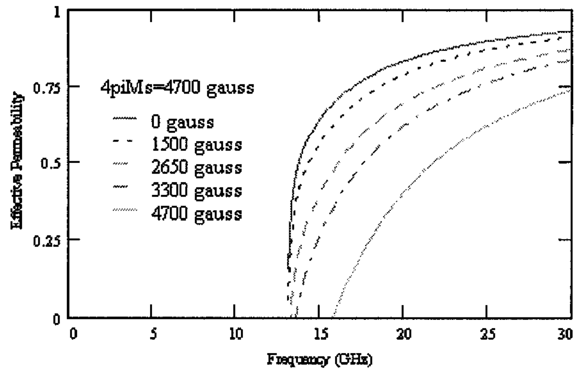


Figure 8. μ_{eff} vs frequency for several values of B_r.

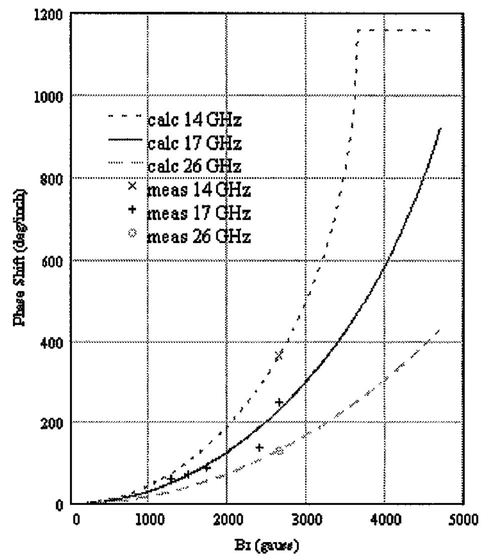


Figure 9. Predicted and measured phase shift per unit length as a function of remanent magnetization B_r, with $4\pi M_s$=4700 gauss.

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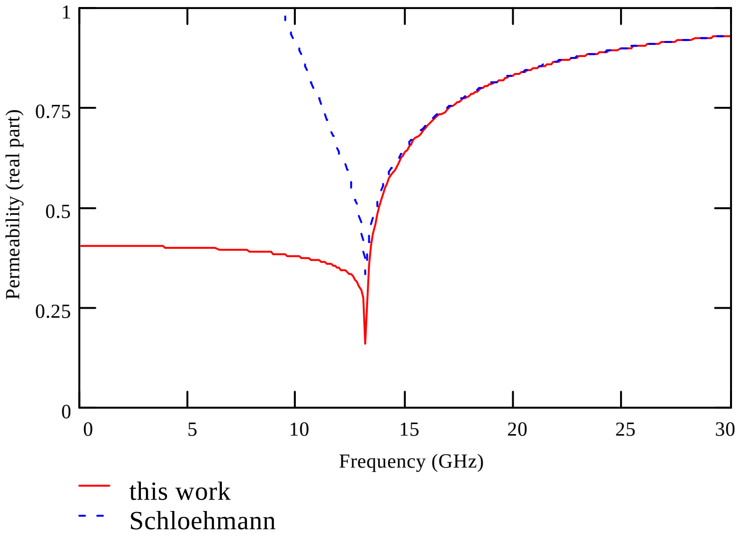
2. J. W. Simon, W. K. Alverson, J. E. Pippin, *A Reciprocal TEM Latching Ferrite Phase Shifter*, International Microwave Symposium, 1966, pp. 241-246.
3. Ernst Schlömann, *Microwave Behavior of Partially Magnetized Ferrites*, J. Appl. Phys. Jan. 1970, pp. 204-214.
4. Jerome J. Green, Frank Sandy, *Microwave Characterization of Partially Magnetized Ferrites*, IEEE Trans MTT June 1974, pp. 641-645.

APPENDIX

Schloemann's[A1] expression for the permeability of a demagnetized substrate is $\mu_{Schl}(f) := \frac{1}{3}\cdot\left[1 + 2\cdot\sqrt{\left|1 - \left(\frac{f_s}{f}\right)^2\right|}\right]$. This gives about the same results as my formula above $\gamma 4\pi M_s$, but does not give the correct insertion phase in the absorption band. In fact, it predicts a phase slope $-d\phi/d\omega<0$, which implies *negative* group velocity. The formulas are compared in figure A1.



The formula given here for the longitudinally magnetized state is somewhat reminiscent of the off-diagonal (κ) portion of an expression proposed by Green, et al[A2] for the tensor permeability elements in the remanent state:

$$\mu := \mu_{dem} + \left(1 - \mu_{dem}\right)\cdot\left(\frac{B_r}{4\cdot\pi\cdot M_s}\right)^{\frac{3}{2}} \qquad \kappa := \frac{\gamma\cdot B_r}{\omega}$$

No physical basis was claimed for this expresssion. The figure below compares the μ_s developed here with Green's effective permeability

$$\mu_{eff} := \frac{\mu^2 - \kappa^2}{\mu}$$